Intelligent Systems: Reasoning and Recognition

James L. Crowley

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Temporal Reasoning - Allen's Temporal Logic

Goals for this lecture:
Illustrate the role of relations in reasoning and knowledge representation
Illustrate an insight about uncertainty as sets of possible answers.

Allen's Temporal Logic

The Temporal Logic of James Allen provides a simple and practical method for reasoning about the temporal relations between events. The system provides an illustration of the role of relations in reasoning and in structured knowledge representation.

Allen's Temporal Logic is designed to express knowledge about temporal relations in a manner that

1) Permits expression of relations that are relative and imprecise.
2) Permits expression and reasoning about uncertainty about the temporal relations between events.
3) Supports reasoning at variable scales of time.
4) Supports persistence

Allen's temporal logic is based on representing events as intervals of time.

Time: an ordering relation on an infinite set of points.
The temporal axis is infinitely dense. Between any two points lies a point.
Interval: An ordered set of points T = {t} defined by "end-points" t^+ and t^-
(t^-, t^+): $(\forall t \in T)$ $(t > t^-)$ and $(t^+ < t)$

For intervals defined in this way, there are 13 possible relations between any pair of intervals. Of these 13 relations, 7 are basic, 6 are inverse relations.
The 7 basic relations includes equals, which is its own inverse.

Temporal relations make it possible to reason about time without knowing the actual time of events. The actual values of (t^-, t^+) are NOT used.

Allen's temporal relations

For the intervals t and s, the 7 basic relations are:

<table>
<tr><th>Name</th><th>Notation</th><th>Schema</th><th>Definition</th><th>Inverse Name</th><th>Inverse Symbol</th></tr>
<tr><td>equal</td><td>t=s</td><td>|-t-|
|-s-|</td><td>$(t^- = s^-) \wedge (t^+ = s^+)$</td><td></td><td></td></tr>
<tr><td>before</td><td>t<s</td><td>|-t-|
|-s-|</td><td>$t^+ < s^-$</td><td>after</td><td>s > t</td></tr>
<tr><td>overlap</td><td>t o s</td><td>|-t-|
|-s-|</td><td>$(t^- < s^-) \wedge (t^+ > s^-) \wedge (t^+ < s^+)$</td><td>overlap inverse</td><td>s oi t</td></tr>
<tr><td>meets</td><td>t m s</td><td>|-t-|
|-s-|</td><td>$t^+ = s^-$</td><td>meets inverse</td><td>s mi t</td></tr>
<tr><td>during</td><td>t d s</td><td>|-t-|
|---s---|</td><td>$(t^- > s^-) \wedge (t^+ < s^+)$</td><td>during inverse</td><td>s di t</td></tr>
<tr><td>starts</td><td>t s s</td><td>|-t-|
|---s---|</td><td>$(t^- = s^-) \wedge (t^+ < s^+)$</td><td>starts inverse</td><td>s si t</td></tr>
<tr><td>finishes</td><td>t f s</td><td>|-t-|
|---s---|</td><td>$(t^- > s^-) \wedge (t^+ = s^+)$</td><td>finish inverse</td><td>s fi t</td></tr>
</table>

<u>Uncertainty</u> in the relative time of two events is represented by listing the possible set of relations between the events.

The set of possible relations between two intervals are represented by a labeled pointer. The label is the set of possible relations.

Whenever a relation is asserted, its inverse is also asserted.

Relations are represented as a labeled arrow: For example, for intervals A, B:

A --{R_{AB}}--> B

For example: of A --{<} ->B or A --{o} ->B or A --{m } ->B
then A --{< o m} ->B

Initially all relations are possible (total uncertainty).
As relations are asserted between intervals, they impose constraints on other relations.

For example

```
A ------{ d}------> B
                    |
                 { > o}
                    |
                    v
                    C
```

Then it must be that R_{AC} = Transitivity(d, >) U Transitivity(d, o)

R_{AC} = {>} ∪ {< o m d s} = {> < o m d s }

When a new relation is asserted between two intervals, it places constraints on all other relations.

For example, if we discover that A ---{ >}---> C
then we can infer that R_{BC} = {< o m d s}
How?

R_{AC} = { > } => R_{CA} = { < }
then

R_{BC} = Transitivity{ R_{CA}, R_{AB} } = T(<, d) = {< o m d s}

A table of transitivity determines constraints between relations

Table of Transitivity

Transitivity is determined by a 12x12 table of transitive relations.
(why 12 and not 13? Ans: "=" is idempotent (i.e. it's own inverse.)

For example : for (A < B) et (B ? C)

	(B ? C)	Constraint on (B ? C)
(A < B)	(B < C)	{<}
(A < B)	(B > C)	No Info
(A < B)	(B d C)	{< o m d s}
(A < B)	(B di C)	{< }
(A < B)	(B o C)	(< }
(A < B)	(B oi C)	{< o m d s}

(A < B) (B m C) {< }
(A < B) (B mi C) {< o m d s}
(A < B) (B s C) {<)
(A < B) (B si C) {< }
(A < B) (B f C) {< o m d s}
(A < B) (B fi C) {<}

The transitivity table is used to develop the set of possible relations between each pair of intervals.

Given A --{R_{AB}}--> B --{R_{BC}}--> C

Possible Relations for A --{R_{AC}}--> C are give by R_{AC} = Transitivity(R_{AB},R_{BC})

Transitivity (R_{AB},R_{BC})
R_{AC} <- NIL;
$\forall\ r_{ab} \in R_{AB}$
$\forall\ r_b \in R_{BC}$
$R_{AC} := R_{AC} \cup T(r_{ab}, r_{BC})$;
RETURN R_{AC};

Constraint Propagation

When a new relation is asserted for two intervals A, B
This new set of relations then imposes constraints on all other relations in the network.

Given A --{R_{AB}}--> B --{R_{BC}}--> C
^------------{$R^B{}_{AC}$}-----------^

however

Given A --{R_{AD}}--> D --{R_{DC}}--> C
^------------{$R^D{}_{AC}$}-----------^

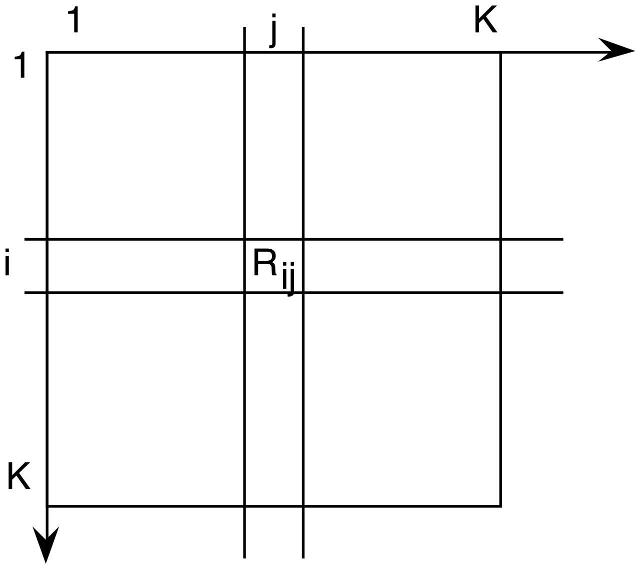
Then $R_{AC} = R^B{}_{AC} \cap R^D{}_{AC}$

It is necessary to propagate constraints for the entire network.

Propagate (interval A)
∀ interval i
∀ interval j
$R_{Ai} := R_{Ai} \cap$ Transitivity (R_{Aj},R_{ji});

There are $(N-1)^2/2$ pairs of intervals.
The algorithmic complexity for N intervals is $O(N^2)$ operations.

Whenever a new relation is asserted, the constraints propagate through the network.
For N intervals, we can see the network as a N x N table of relations
Each new entry requires visiting all cells in the table $O(N^2)$ operations.



For example; if we assert that the possible relations between A and B are the list $NewR_{AB}$, then we replace R_{AB} by $NewR_{AB}$.

This can cause a problem as the set of intervals grows.
To avoid a combinatoric explosion, Allen proposed to group sets of relations into reference intervals. Constraints are propagated only between the intervals within the same reference.

for example : Hour, Day, Week, Month and Year are all reference intervals.

Reference Intervals.

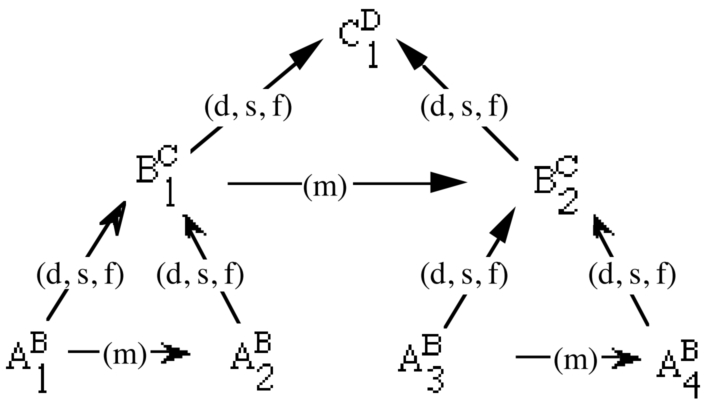
A reference interval is a set of intervals with a complete graph of temporal relations between them.

No explicit relations are made with intervals outside the reference. Instead, the network of relations between the references is used to determine relations when needed.

Reference intervals form a hierarchy.

Notation : A_k^B : Interval A_k for the reference B.

Example :



If two intervals are not directly related, then their relations are determined by ascending the tree.

Exercise:

Assume the following temporal relations between intervals A, B, C and D.

Event A meets Event B :	(A m B)
Event B meets event C :	(B m C)
Event D is after event A:	(D > A)
Event D is before event C :	(D < C)

a) What are the possible relations of D to B obtained by transitivity with A?

T(D > A, A m B) = { d, f, oi, mi, >}

b) What are the possible relations of D to B obtained by transitivity with C?

T(D < C, C mi B) = {<, o, m, d, s}

c) What are the possible relations of D to B after constraint propagation?

(D d B)