Computer Vision

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Lesson 6 — Exercises

For an image *P(i,j)*, the gradient $\vec{\nabla}P(i,j)$ is a vector composed of the first derivatives in the row and column directions. In the following questions, assume that *P(i,j)* is a luminance (black and white) image of size 1024 x 1024 pixels with 8 bits per pixel.

1) You are asked to compute the gradient $\vec{\nabla}P(i,j)$ using convolution with sampled Gaussian derivatives, $\vec{\nabla}G(i,j)$. Give the formulae for the sampled Gaussian derivatives as well as the formulae for the 2D convolution.

2) What is the minimum size support window that can be used for a Gaussian derivative with σ=2? What is the computational cost for such a convolution in terms of additions and multiplications when implemented as a 2-D convolution?

3) Show that the sampled Gaussian derivatives are separable. That is, that convolution with the sampled Gaussian derivative can be implemented as a sequence of convolutions with 1-D filters in the row and column directions. What is the computational cost in terms of additions and multiplications for the convolution of the image with sampled Gaussian derivatives at σ=2 when implemented as convolution with separable 1D components?

4) Show that a 1D Gaussian low pass filter with σ=2 can be implemented as a series of convolutions with 1D Gaussian low pass filter with σ=1. How many convolutions are needed to compute a 1D Gaussian low pass filter with σ=2 as a series of convolutions with a 1D Gaussian low pass filter with σ=1? What is the computational cost of this series of convolutions?

5) Is it possible to implement the convolution with a 1-D Gaussian Derivative filter with σ=2 as a series of convolutions with 1-D Gaussian filters? If yes, how? If no, why not?

6) Given the Gradient of the image $\vec{\nabla}P(i,j)$ in the row and column directions, give a formulae to determine the gradient at pixel (i,j) in an arbitrary direction θ.

7) Given the Gradient of the image $\vec{\nabla}P(i,j)$, give the formulae to determine the direction of maximum gradient, θ_{max}(i,j) at each pixel i,j.

8) Give a formula for the coefficients for a binomial low pass filter at σ=2.

9) Show that a binomial low pass filter at σ=2 can be implemented as a series of convolutions with the binomial filter [1 2 1]. How many such convolutions are required?

10) Assume that you use [1 0 -1] as a first derivative operator. What is the computational cost (additions and multiplications) for computing the Gradient of an image at σ=2 using a series of convolutions with 1-D binomial filters [1 2 1] in the row and column directions?