Intelligent Systems: Reasoning and Recognition

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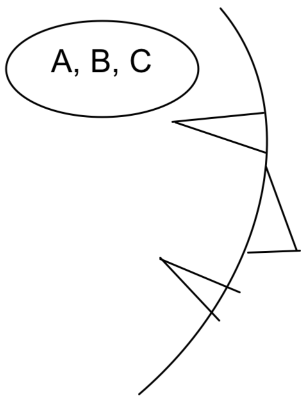
Problem Solving and Reasoning Under Uncertainty

Edwin T. Jaynes, "Probability Theory: The Logic of Science", Edited by G. L. Bretthorst, Cambridge University Press 2003.

Copies of selected chapters from (Jaynes 03) can be found on the course web site.

The Intelligent Agent

To provide a formal basis for studying intelligence, in 1970s, Nils Nilsson proposed the Intelligent Agent as a fundamental concept for formalizing intelligence.



The Intelligent Agent is an abstract concept composed of 3 pars: (A, B, C)
A) Actions; The ability to act; A physical body;
B) Goals. (In French "Buts")
C) Knowledge; The ability to choose actions to accomplish goals.

The "Intelligent Agent" is "rational". That it, the intelligent agent chooses its actions to accomplish its goals. This is called the "Principal of Rationality".

Nilsson proposed to define intelligence as the ability to choose actions to accomplish goals. Newell adopted this idea and in his 1980 address accepting the Turing award, Newell proposed that "knowledge" is anything that can be used by an agent to choose actions in order to accomplish goals.

Rational Intelligence: The ability to choose actions to accomplish goals.

An agent is intelligent if it 1) can act, 2) has goals, and 3) Can choose its actions to accomplish it's goals.

Rational intelligence leads to a formulation of intelligence as problem solving and planning, formalized using the notion of state space.

Planning and Problem Solving

Planning: The search for a sequence of actions leading to a goal.

Rationality leads to a formulation of intelligence as planning
Rational intelligence is formalized using a Problem space.

A problem space is defined as
1) A set of states {U},
2) A set of operators for changing states {A} (Actions).

A state is defined using a conjunction of predicates. (an inclusion test).

A problem is {U}, {A} plus
an initial state $i \in \{U\}$
a set of Goal States $\{G\} \subset \{U\}$

A plan creates a sequence of actions A_1, A_2, A_3, A_4,... that lead from the state S to one of the states $g \in \{G\}$

States: A state, s, is a "partial" description of the real universe.

A state is defined as a conjunction of predicates (Truth functions) based on measured (observed) values. The measured values are called "observations".

Examples:
Mobile Robotics: Near(x, y, t)
Blocks World: OnTable (A) ∧ On (A, B) ∧ HandEmpty

The external universe is described as states defined as logical assemblies of predicates concerning observations.

Planning as Search

Planning is the generation of a sequence of actions to transform i to a state $g \in \{G\}$
Planning requires search for a path through a graph of states. .

The "paradigm" for planning is "Generate and Test".

Given a current state, s
1) Generate all neighbor states {N} reachable via 1 action.
2) For each $n \in \{N\}$ test if $n \in \{G\}$. If yes, exit
3) Select a next state, $s \in \{N\}$ and loop.

Depending on step 3, the search can be :

1) Depth first search
2) Breadth first search
3) Heuristic Search
4) Hierarchical Search

Depth-First, Breadth First and Heuristic Search unified within the GRAPHSEARCH algorithm of Nilsson. Nilsson attempted to fomalise Hierachical search with a family of algorithms.

Searching a graph has exponential algorithm complexity.
"knowledge" can be used to reduce the complexity.

Under certain conditions, heuristic search can be said to be "optimum". In this case it is said to be A*

Problems:
1) The Universe is open (infinitely complex). The number of possible states is infinite. (It is always possible to define new predicates, and hence new states).
thus
2) Knowledge of the universe is necessarily incomplete.
and even more
3) Knowledge of the universe can be in error and is never certain.

Reasoning under Uncertainty

Knowledge: the ability to choose actions to accomplish goals.

Understanding: the ability to predict and explain phenomena

Phenomena : Anything that can be observed.

Reasoning: the ability to generate new knowledge.

Understanding enables reasoning

We know of several "kinds" of reasoning.

Deductive Reasoning: A process of logical inference in which a conclusion in which a proposition is found to be logically consistent (true) with a set of axioms and postulates.

Abductive Reasoning: a form of logical inference in which a hypothesis is formulated to account for an observation. Unlike deductive reasoning, the resulting proposition is uncertain.

Inductive Reasoning: The ability to generate general statements from specific observations. Inductive reasoning is inherently uncertain. It deals with degrees of certainty, that increase with accumulation of evidence. Inductive reasoning allows for the possibility of that the conclusion is false. (note that this is different from induction in mathematics)

Because the universe is infinite, and because our observations can contain errors (creating false evidence), Deductive Reasoning is not sufficient for general intelligence.

A Brief Review of First order Predicate Calculus

The first order Predicate calculus is a deductive reasoning system that can be used to determine if a proposition, expressed as a logical statement, is logically consistent with a set of Propositions and Axioms.

Axiom: A statement whose truth is self-evident (immutable).
Postulate: A statement that is assumed to be true
Proposition: A statement whose truth is to be tested.

First-order logic is the standard formal logic for axiomatic systems. While logic dates back to Aristotle, much of the modern foundation was established by George Boole. The first order predicate calculus was developed by Bertrand Russel based on the work of Gottlob Frege and C. S. Pierce.

A First order logic manipulates symbols without reference to their meaning.
Logic is a purely syntactic shallow reasoning system.

A 1st order logic is defined as a set of Symbols, Variables, Predicates, and Logic operators.

1) Symbols: S = {A, B, C, … } (the domain of discourse).

A set of symbols provide the "domain of discourse" for a logic system. We will label symbols with capital letters. Symbols denote phenomena (observations) from outside the logical system. We will use upper case letters for symbols.

If the number of symbols is finite and known in advance, then the system is referred to as a "closed" system. A number of simplifications are possible in closed systems. Unfortunately most real world problems require an unbounded set of symbols.

Symbols can be collected into sets, either by extension (Listing the members of the set) or intention (defining a test for membership). Sets of symbols can be represented by quantified variables, represented as lower case letters (ex: x, y, z, w).
Variables can be defined to represent sets of symbols. We will represent variables with lower case letters. For example, we can define x = {A, B, C}. The set can be intentional (listed)

2) Variables and Quantifiers: ∀x, ∃y:

The possible values of a variable representing a can be enumerated using one of two quantifiers: ∀x and ∃x. The quantified variable is defined only within the scope of the statement.

For All x - ∀x:(-)
For Each x- ∃x:(-)

3) <u>Predicate Functions</u>: Predicates are truth functions, that map symbols into truth values. In 1st order predicate calculus, predicates are Boolean and can be only TRUE or FALSE. These values are also represented by the symbols T, F as well as 1, 0 etc)

We will use lower case functions to define predicates function. p(), q(A), r(x, B).

The number of arguments of a predicate is the "arity". Predicates can be defined with an arity of any natural number including 0.

Notation: When a predicate has no arguments (i.e. when predicates of arity 0), we may sometimes drop the parentheses as a notational convenience.
For example *p() == p*

Do not confuse the predicate "*p*" with a variable "*x*". This should be evident from the use.

Note that only symbols or variables can be arguments of a predicate. A predicate of a predicate would be a "Second Order" predicate. Higher order predicates must be used very carefully as they can rapidly lead to contradictions.

An example of a Second Order is given by Modal Logic. A modal logic adds the second order predicates "Necessary" "Possible":

"p() is necessary" is written □p()
"p() is possible" is written ◊p().

Modal operators were defined by C.I. Lewis in 1932.

Lewis, C. I. and C. H. Langford, "Symbolic Logic", Dover Press, NY, 1932

A formal, correct semantic for modal logic was provided using the "possible worlds" concept by Kripke in 1961. This requires an operator for changing "worlds".

Kripke, S. "Semantical Considerations on Modal Logic", in Proof Methods for Modal and Intuitionistic Logics, edited by M. Fitting, Reidel, Boston, MA 1983.

A predicate is necessary if it is true in all adjacent worlds.
A predicate is possible if it is true in at least one adjacent worlds.

Modal logics are interesting by they are not sufficient for reasoning under uncertainty.

4) Logical operators: Predicates can be combined with logical operators to form statements. The basic set of operators are AND, OR, NOT. We can define a minimal set using NOT plus either AND or OR. However, generally it is convenient to use all three as this will lead to simpler and more easily read statements.

Note that operators can only be applied to predicates, and NEVER to symbols or to variables. Example of symbols used for logical operators include:

AND	∧	·	∧
OR	∨	+	∨
NOT	¬	over-bar	~

For example p() AND q() OR NOT r() = $p()\wedge q()\vee\neg r = pq+\bar{r}$

Operators are defined by Truth tables, as shown by Ludwig Wittgenstein in:
Tractatus Logico-Philosophicus, 1921.

Wittgenstein was student of Bertrand Russel in Cambridge from 1910 to 1914.
He wrote the notes for the Tractatus while he was a German soldier during World War I and completed it when a prisoner of war at Como and later Cassino in August 1918.

Let us use T for True and F for False. The meaning of the operators ∧, ∨, ¬ are:

p()	¬p()
T	F
F	T

p	q	p∧q	p∨q
T	T	T	T
T	F	F	T

F	T	F	T
F	F	F	F

Operators are combined with predicates to define "statements".
Both postulates and propositions are statements composed of predicates connected with operators. In 1st order predicate calculus we seek to determine the logical consistency of a statement (proposition) with a set of statements (axioms and propositions).

Common algebraic properties used with statements include:

Idempotence: $p \wedge p = p$
$p \vee p = p$

Communativity: $p \wedge q = q \wedge p$
$p \vee q = q \vee p$

Associativity: $p \wedge (q \wedge r) = (p \wedge q) \wedge r = p \wedge q \wedge r$
$p \vee (q \vee r) = (p \vee q) \vee r = p \vee q \vee r$

Distributivity: $p \wedge (q \vee r) = (p \vee q) \wedge (p \vee r)$

Duality: IF $p = q \wedge r$ THEN $\neg p = \neg q \vee \neg r$

(equivalence) IF $p = q \vee r$ THEN $\neg p = \neg q \wedge \neg r$
IF $\forall x: \neg p(x)$ THEN $\neg \exists x: p(x)$

Note that the quantifiers are redundant. We could also have written:

$$\forall x: \neg p(x) \Leftrightarrow \neg \exists x: p(x)$$

It is possible to use truth tables to define other operators. For example:

Implies	$\rightarrow$
Equivalence	$\leftrightarrow$
Meta Implication	$\Rightarrow$
Meta Equvalence	$\Leftrightarrow$

Implication:

Consider $p() \rightarrow q()$

For humans, when $p \rightarrow q$ then
1) if p is true then q is true. However, $(p \rightarrow q) \wedge p \Rightarrow q$
2) if p is false then q is may be true or false. $(p \rightarrow q) \wedge \neg p \Rightarrow ?$

There can be other causes of q.
if Joe has the flu then Joe has a headache
if Joe does not have the flu, joe may still have a headache.

Similarly, if we know $p \rightarrow q$ and we know $\neg q$, then we can know $\neg p$
$(p \rightarrow q) \wedge \neg q \Rightarrow \neg p$

However, If we know that $p \rightarrow q$ and we know q then this tells us nothing about p
$(p \rightarrow q) \wedge q \Rightarrow ?$

In predicate calculus, implication is given a strict meaning that does not correspond to human intuition.

$p \rightarrow q \Leftrightarrow p \vee \neg q$ as defined by the following truth table:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Thus if q is false then p must be also be false.
This notion of "implication" in first order predicate calculus is not consistent with human common sense.

Deductive Reasoning and Uncertainty

A "contradiction" is a demonstration that a statement is both true and false.

In a 1st order predicate calculus, if the proposition is not consistent with any of the postulates, then it will lead to a contradiction. Similarly, if any of the postulates are inconsistent, then any proof can lead to a contradiction.

Most automated theorem-provers, such as Greens Theorem used in prolog, use this to provide by "proof by the absurd".

A proposition is tested to see if it leads to a contradiction. If a proposition does not lead to a contradiction, then it is consistent with the propositions and considered to be TRUE.

As a result, a 1st order predicate calculus can NOT reason with uncertain or incorrect statements in either its postulates or its propositions as any error leads to a contradiction.

Unfortunately, in the real world:

Our perception of the universe is necessarily incomplete. (universe is open)
Our perception of the universe is never certain. (errors in perception)
It is common to perceive in-consistent "facts" .

Thus predicate calculus is not sufficient as a general purpose reasoning tool.