Computer Vision

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Lesson 4 — Exercises

For an image *P(i,j)*, the gradient $\vec{\nabla} P(i,j)$ is a vector composed of the first derivatives in the row and column directions. In the following questions, assume that *P(i,j)* is a luminance (gray-scale or black and white) image of size 1024 x 1024 pixels with 8 bits per pixel.

1) You are asked to compute the gradient $\vec{\nabla} P(i,j)$ using convolution with sampled Gaussian derivatives, $\vec{\nabla} G(i,j)$. Give the formulae for the sampled Gaussian derivatives as well as the formulae for the 2D convolution.

2) What is the minimum size support window that can be used for a Gaussian derivative with $\sigma=2$? What is the computational cost for such a convolution in terms of additions and multiplications when implemented as a 2-D convolution?

3) Show that a 1-D Gaussian low pass filter with $\sigma=2$ can be implemented as a series of convolutions with 1-D Gaussian low pass filter with $\sigma=1$. How many convolutions are needed to compute a 1D Gaussian low pass filter with $\sigma=2$ as a series of convolutions with a 1D Gaussian low pass filter with $\sigma=1$? What is the computational cost of this series of convolutions?

4) Show that the sampled Gaussian derivatives can be implemented as a sequence of convolutions with 1-D filters in the row and column directions. What is the computational cost in terms of additions and multiplications for the convolution of the image with sampled Gaussian derivatives at $\sigma=2$ when implemented as convolution with separable 1D components?

5) Given the Gradient of the image $\vec{\nabla} P(i,j)$, give the formulae to determine the direction of maximum gradient, $\theta_{max}(i,j)$ at each pixel i,j.

6) Given the Gradient of the image $\vec{\nabla} P(i,j)$ in the row and column directions, give a formulae to determine the gradient at pixel (i,j) in an arbitrary direction θ.