Computer Vision

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Lesson 2

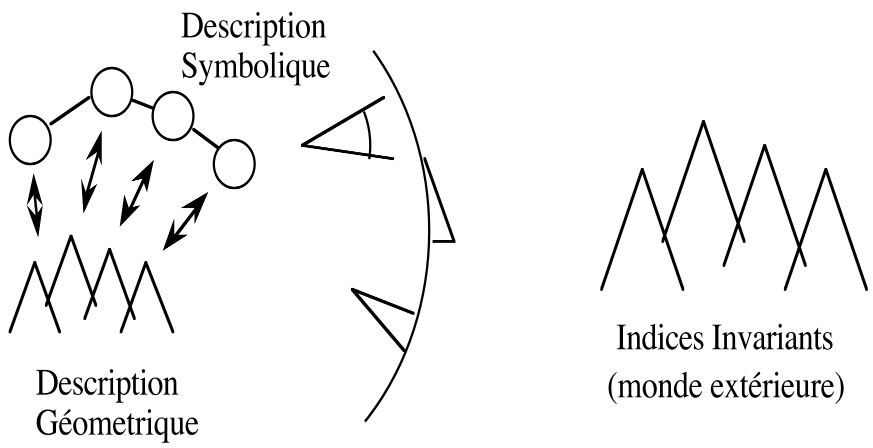
Description of Image Contrast

Lesson Outline:

1 Describing Contrast

An image is simply a large table of numerical values (pixels).
The "information" in the image may be found in the colors of regions of pixels, and the variations in intensity of pixels (contrast).

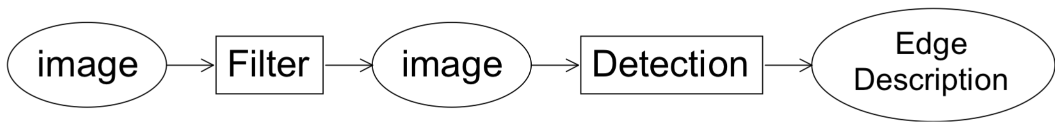
Extracting information from an image requires organizing these values into patterns that are "invariant" to changes in illumination and viewing direction.



Color provides information about regions of constant pigment.
Contrast provides information about 3D shape, as well as surface markings.

Contours of high contrast are referred to as "edges".

Edge detection is typically organized in two steps
1) contrast filtering
2) edge point detection and linking.



Two classic contrast detection operators are:

1) Roberts Cross Operator, and
2) The Sobel edge detector.

2 Color Invariance

Color Constancy: The subjective perception of color is independent of the spectrum of the ambient illumination.

Subjective color perception is provide by "Relative" color and not "absolute" measurements.

This is commonly modeled using a Color Opponent space.

The opponent color theory suggests that there are three opponent channels: red versus green, blue versus yellow, and black versus white (the latter type is achromatic and detects light-dark variation, or luminance).

This can be computed from RGB by the following transformation:

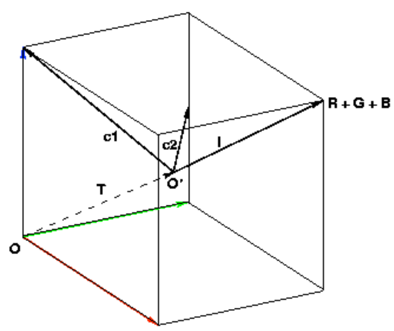
Luminance : L = R+G+B

Chrominance: C1 = (R-G)/2

C2 = B – (R+G)/2

as a matrix :

$$\begin{pmatrix} L \\ C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ -0.5 & -0.5 & 1 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$



Such a vector can be "steered" to accommodate changes in ambient illumination.

3 Describing Image Contrast

3.1 Roberts Cross Edge Detector

One of the earliest methods fore detecting image contrast (edges) was proposed by Larry Roberts in his 1962 Stanford Thesis.

Note, in this same thesis, Roberts introduced the use of homogeneous coordinates for camera models, as well as wire frame scene models. Roberts subsequently went to work for DARPA where he managed the program that created the Arpanet (now known as the internet).

Roberts Cross operator employs two simple image filters:

$$m_1(i,j) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad m_2(i,j) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

These two operators are used as filters. They are convolved with the image.

Convolution (or filtering) : for n = 1, 2

$$E_n(i, j) = m_n * p(i,j) = \sum_{k=0}^{1}\sum_{l=0}^{1} m_n(k,l)p(i-k, j-l)$$

The contrast is the module of each pixel :

$$E(i,j) = \left\| \vec{E}(i,j) \right\| = \sqrt{E_1(i,j)^2 + E_2(i,j)^2}$$

The direction of maximum contrast is the phase

$$\varphi(i,j) = Tan^{-1}\left(\frac{E_2(i,j)}{E_1(i,j)} \right) + \frac{\pi}{4}$$

Because of its small size and simplicity, the Roberts detector is VERY sensitive to high spatial-frequency noise. This is exactly the noise that is most present in images.

To reduce such noise, it is necessary to "smooth" the image with a low pass filter.
We can better understand the Roberts operators by looking at their Fourier Transform.

$$M_n(u,v) = \sum_{k=0}^{1}\sum_{l=0}^{1} m_n(k,l) e^{-j(ku+lv)}$$

$$M_1(u,v) = (+1)\cdot e^{-j(-(0)u+-(0)v)} + (-1)\cdot e^{-j(u+v)} = 2j\ Sin(0.5u+0.5v)$$

$$M_2(u,v) = (+1)\cdot e^{-j(-(0)u+-(1)v)} + (-1)\cdot e^{-j(-(1)u+-(0)v)} = 2j\ Sin(0.5u-0.5v)$$

3.2 The Sobel Detector

Invented by Irwin Sobel in his 1964 Doctoral thesis, this edge detector was made famous by the the text book of R. Duda adn P. Hart published in 1972.

It is perhaps the most famous and widely used edge detector:

$$m_1(i,j) = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} \qquad m_2(i,j) = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

Convolution (or filtering) : for n = 1, 2

$$E_n(i, j) = m_n * p(i,j) = \sum_{k=-1}^{1}\sum_{l=-1}^{1} m_n(k,l)\,p(i-k, j-l)$$

The contrast is the module of each pixel :

$$E(i,j) = \left\|\vec{E}(i,j)\right\| = \sqrt{E_1(i,j)^2 + E_2(i,j)^2}$$

The direction of maximum contrast is the phase

$$\varphi(i,j) = Tan^{-1}\left(\frac{E_2(i,j)}{E_1(i,j)}\right)$$

Sobel's Edge Filters can be seen as a composition of a image derivative and a smoothing filter.

$$m_1(i,j) = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$m_2(i,j) = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

The filter $\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$ is a form of image derivative.

The filter $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$ is a binomial smoothing filter.

3.3 Difference Operators: Derivatives for Sampled Signals

For the function, s(x) the derivative can be defined as :

$$\frac{\partial s(x)}{\partial x} = \lim_{\Delta x \to 0}\left\{\frac{s(x+\Delta x) - s(x)}{\Delta x}\right\}$$

For a sampled signal, s(n), an the equivalent is $\frac{\Delta s(n)}{\Delta n}$

the limit does not exist, however we can observe

$$\Delta \text{n} = 1 : \quad \frac{\Delta s(n)}{\Delta n} = \frac{s(n+1) - s(n)}{1} = s(n) * \begin{bmatrix} -1 & 1 \end{bmatrix}$$

$$\Delta \text{n} = 0 : \quad \frac{\Delta s(n)}{\Delta n} = \frac{0}{0}$$

This is the operator used by Roberts.

If we use a Symmetric definition for the derivative:

$$\frac{\partial s(x)}{\partial x} = \lim_{\Delta x \to 0}\left\{\frac{s(x+\Delta x) - s(x-\Delta x)}{\Delta x}\right\}$$

then

$$\Delta \text{n} = 1 : \quad \frac{\Delta s(n)}{\Delta n} = \frac{s(n+1) - s(n-1)}{1} = s(n) * \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

This is the operator used by Sobel.

Note that a derivative is equivalent to convolution!
We can define derivation in the fourier domain as follows:

$$F\left\{\frac{\partial s(x)}{\partial x}\right\} = -j\omega \cdot F\{s(x)\}$$

and thus

$$\frac{\partial s(x)}{\partial x} = F^{-1}\{-j\omega\} * s(x)$$

If we can determine $d(x) = F^{-1}\{-j\omega\}$ then we have our derivative operator. If we "sample" d(x) to produce d(n) we have a sampled derivative operator.

Unfortunately, $F^{-1}\{-j\omega\}$ has an infinite duration in x, and thus d(n) is an infinite series. However, the first term of d(n) is [-1 0 1].

Thus we can define the first "difference" operator as a first order approximation for the derivative of a discrete signal.

$$\Delta_i p(i,j) = \Delta p(i,j)/\Delta i = p(i,j) * [-1, 0, 1]$$

$$\Delta_i p(i,j) = \frac{\Delta p(i,j)}{\Delta i} = p(i,j) * \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

$$\Delta_j p(i,j) = \frac{\Delta p(i,j)}{\Delta j} = p(i,j) * \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

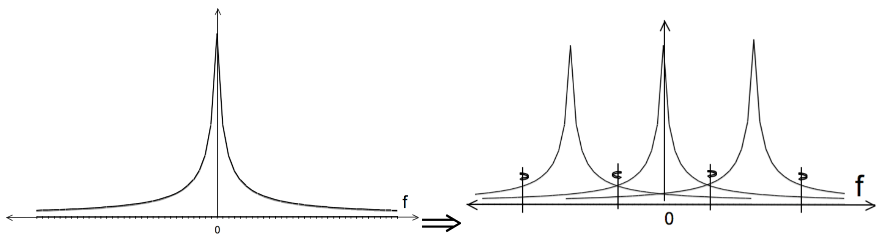
$$\nabla P(i,j) = \begin{pmatrix} \Delta_i p(i,j) \\ \Delta_j p(i,j) \end{pmatrix}$$

$$E(i,j) = \left\| \nabla P(i,j) \right\|$$

$$\vartheta(i,j) = Tan^{-1}\left(\frac{\Delta_j p(i,j)}{\Delta_i p(i,j)} \right)$$

This works fine, except that such a derivative operator amplifies sampling noise.

When a signal s(x) is sampled to create s(n), sampling noise is introduced



Sampling adds repeated copies of the spectrum at periods of two times the nyquist frequency $2F_n = 2/T$. The result amplifies high frequency noise.

The first difference filter $d_1(n)$ = [1, 0, -1] has a Fourier transform:

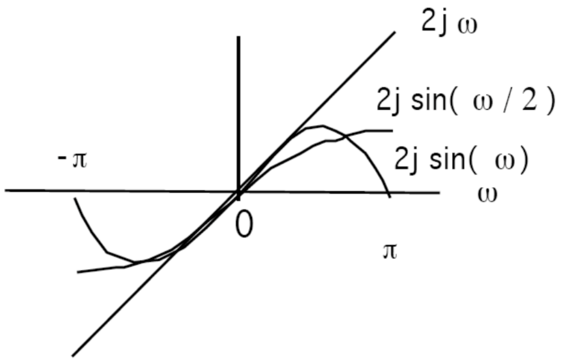
$$D(\omega)= \sum_{n=-1}^{1} d(n)e^{-j\omega n}$$
$$D(\omega)= 1e^{-j\omega(-1)} + 0e^{-j\omega 0} + (-1)e^{-j\omega(1)}$$
$$D(\omega)= e^{j\omega} - e^{-j\omega}$$
$$D(\omega)= -2j\sin(\omega)$$

Calculation of a derivative is the same as convolution with the filter [1, 0, -1], which is the same as multiplication of the spectrums.

$$d(n) * s(n) \Leftrightarrow D(\omega) \cdot S(\omega)$$

The filter d(n) = [1, -1] is even worse. Its Fourier transform is

$$D(\omega)= -2j\sin(\omega/2)$$



Sobel uses the optimal local derivative filter.

4 Describing Contrast (Continued)

4.1 Difference Operators: Derivatives for Sampled Signals

For the function, s(x) the derivative can be defined as :

$$\frac{\partial s(x)}{\partial x} = \lim_{\Delta x \to 0}\left\{\frac{s(x+\Delta x) - s(x-\Delta x)}{\Delta x}\right\}$$

For a sampled signal, s(n), an the equivalent is $\frac{\Delta s(n)}{\Delta n}$

the limit does not exist, however we can observe

$$\Delta n = 1 : \quad \frac{\Delta s(n)}{\Delta n} = \frac{s(n+1) - s(n-1)}{1} = s(n) * \begin{bmatrix}-1 & 0 & 1\end{bmatrix}$$

This is the operator used by Sobel.

Thus we can define the first "difference" operator as a first order approximation for the derivative of a discrete signal.

$$\Delta_i p(i,j) = \Delta p(i,j)/\Delta i = p(i,j) * [-1, 0, 1]$$

$$\Delta_i p(i,j) = \frac{\Delta p(i,j)}{\Delta i} = p(i,j) * \begin{bmatrix}-1 & 0 & 1\end{bmatrix}$$

$$\Delta_j p(i,j) = \frac{\Delta p(i,j)}{\Delta j} = p(i,j) * \begin{bmatrix}-1 \\ 0 \\ 1\end{bmatrix}$$

$$\nabla P(i,j) = \begin{pmatrix}\Delta_i p(i,j) \\ \Delta_j p(i,j)\end{pmatrix}$$

$$E(i,j) = \left\|\nabla P(i,j)\right\|$$

$$\vartheta(i,j) = Tan^{-1}\left(\frac{\Delta_j p(i,j)}{\Delta_i p(i,j)}\right)$$

This works fine, except that such a derivative operator amplifies sampling noise.

4.2 Smoothing: The Binomial Low pass filter.

Sobel uses a filter [1, 2, 1] to smooth. This is also an optimal filter.
It is part of a family of filters generated by the binomial series.

The binomial series is the series of coefficients of the polynomial:

$$(x+y)^n = \sum_{m=0}^{n} b_{m,n} x^{n-m} y^m$$

The coefficients can be computed as $b_{m,n} = b_n(m) = [1, 1]^n$

These are the coefficients of Pascal's Triangle.

Les coefficients du suite binomial sont générés par le triangle de Pascal :

n	sum = 2^n	$\mu = n/2$	$\sigma^2 = n/4$	$\sigma = \sqrt{n/2}$	Coefficients
0	1	0	0	0	1
1	2	0.5	0.25		1 1
2	4	1	0/5		1 2 1
3	8	1.5	0.75		1 3 3 1
4	16	2	1	1	1 4 6 4 1
5	32	2.5	1.25		1 5 10 10 5 1
6	64	3	1.5		1 6 15 20 15 1
7	128	3.5	1.75		1 7 21 35 35 21 7 1
8	256	4	2	$\sqrt{2}$	1 8 29 56 70 56 29 8 1

These coefficients provide a family of low pass filters with remarkable properties.
Notably, these are the best approximation for a Gaussian filter of finite extent.
They also happen to have integer coefficients.

$$b_n(m) = b_1(m)^{*n} = [1, 1]^{*n} = \text{n convolutions of } [1, 1]$$

Gain : $s_n = \sum_{m=1}^{n} b_n(m) = 2^n$

Center of gravity is

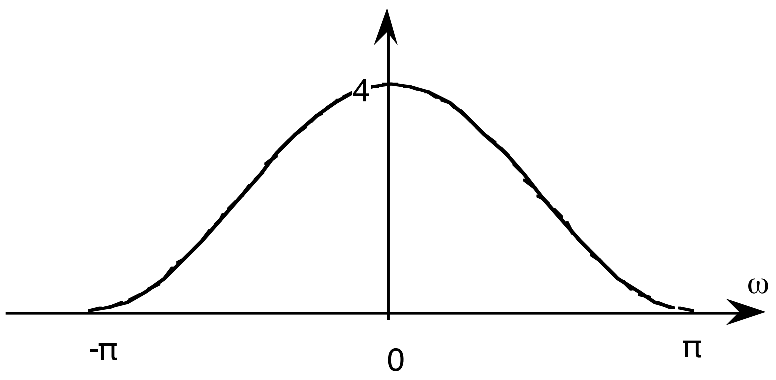
$$\mu_n = \frac{1}{s_n} \sum_{m=1}^{n} b_n(m) \cdot m = \frac{n}{2}$$

The variance is:

$$\sigma_n^2 = \frac{1}{s_n}\sum_{m=1}^{n} b_n(m)\cdot(m-\mu)^2 = \frac{n}{4}$$

The Fourier transform for $b_2(m) = [1, 2, 1]$ is

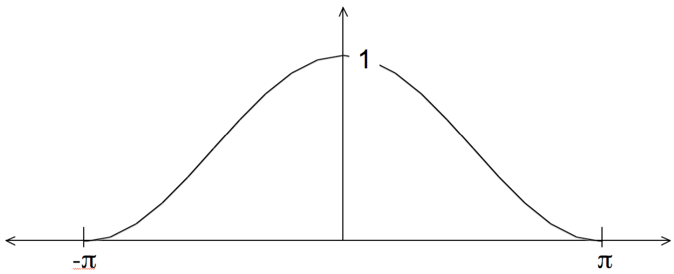
$$B_2(\omega) = \sum_{m=-1}^{1} b_2(m)e^{-j\omega m}$$
$$B_2(\omega) = 1e^{-j\omega(-1)} + 2e^{-j\omega 0} + 1e^{-j\omega(1)}$$
$$B_2(\omega) = 2 + e^{j\omega} + e^{-j\omega}$$
$$B_2(\omega) = 2 + 2\cos(\omega)$$



If we normalize the gain: $b_2(m) = (1/4)[1, 2, 1]$

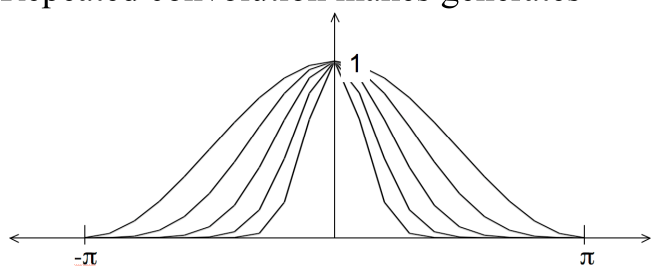
$$B_2(\omega) = \frac{1}{2} + \frac{1}{2}\cos(\omega)$$

Which is a cosine on a platform



$$B_2(\omega) = \frac{1}{2} + \frac{1}{2}\cos(\omega)$$

Repeated convolution makes generates



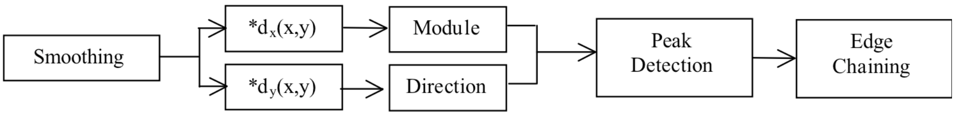
$$B_n(\omega) = \left(\frac{1}{2} + \frac{1}{2}\cos(\omega)\right)^{n/2}$$

The binomial coefficients provide a series of low pass filters with no ripples.

In 2D, the filters provide separable filters that are nearly circularly symmetric

$$\text{2-D} \quad b_2(i, j) = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

4.3 Edge Detection using integer coefficient filters



$$\nabla P(i, j) = \begin{bmatrix} \dfrac{\Delta p(i,j)}{\Delta i} \\ \dfrac{\Delta p(i,j)}{\Delta j} \end{bmatrix} = \begin{bmatrix} m_{1*} p(i,j) \\ m_{2*} p(i,j) \end{bmatrix} = \begin{bmatrix} E_1(i,j) \\ E_2(i,j) \end{bmatrix}$$

Gradient:

$$E(i,j) = \left\| \vec{E}(i,j) \right\| = \sqrt{E_1(i,j)^2 + E_2(i,j)^2}$$

Direction of maximum contrast

$$\varphi(i,j) = Tan^{-1}\left(\frac{E_2(i,j)}{E_1(i,j)} \right)$$

Steps:
1) Smoothing - Suppress high frequency noise
2) Gradient - Compute first derivatives in row and column
3) Detection - Non-maximum suppression with double threshold
4) Chaining - Assembly of connected points above threshold. Elimination of chains where one of the points is not above a second threshold.
5) Polygonal approximation (multiple algorithms exist).

4.4 Non-maximum suppression.

Contrast points are local maxima in E(i, j).

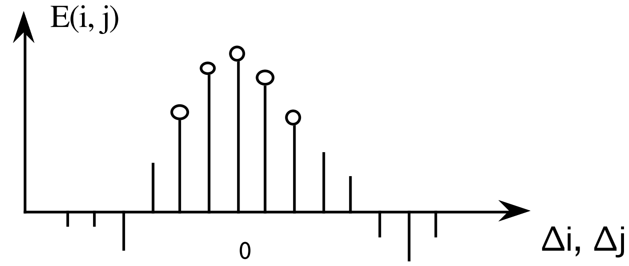
les <u>points</u> de contraste : C(i, j)
pour le gradient de la magnitude E(i, j) et orientation Φ(i, j)

For each point :

1) Determine the direction of maximum gradient:

$$\Delta i = \frac{\Delta_i P(i,j)}{\|\nabla P(i,j)\|} \qquad \Delta j = \frac{\Delta j P(i,j)}{\|\nabla P(i,j)\|}$$

2) Compare the gradient to its neighbors in this direction.



$$c(i,j) = \begin{cases} E(\text{i,j}) & \text{if } E(i-\Delta i, j-\Delta j) \le E(i,j) \ge E(i+\Delta i, j+\Delta j) \\ 0 & \text{Otherwise} \end{cases}$$

Construct a list of connected points for which E(i,j) ≠ 0.

<u>Techniques</u>:

1) Line scan edge chaining algorithm
2) Edge following
3) Hough Transform

5 Hough Transform

The Hough transform is an "optimal" statistical detector for estimating parametric functions from discrete samples. This method was invented for interpreting bubble chamber images in particle physics. It is based on "voting" for possible parameters.

This transform was invented by
P.V.C. Hough, Machine Analysis of Bubble Chamber Pictures, Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

It was patented in a crude form by IBM in 1962 using $y = mx+c$.

It was made popular by Duda and Hart :
Duda, R. O. and P. E. Hart, "Use of the Hough Transformation to Detect Lines and Curves in Pictures," Comm. ACM, Vol. 15, pp. 11–15 (January, 1972)

Consider the line equation

$$x \cos(\theta) + y \sin(\theta) + c = 0$$

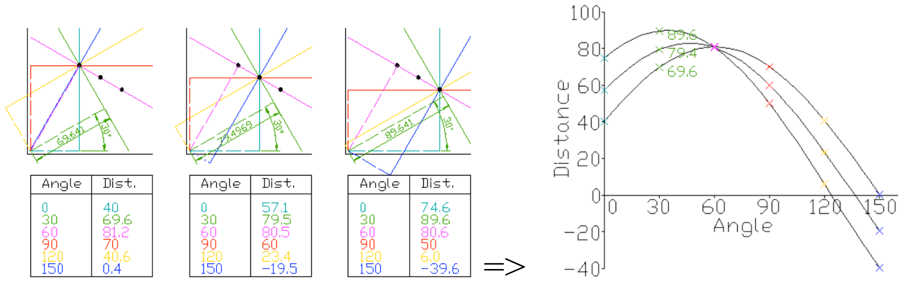
In the image, for each x,y (free parameters) we need to determine (c, θ)

In the Hough transform, we will create a dual space in which (c, θ) are free parameters.
We will estimate lines as peaks in this dual space. To find peaks we build an accumulator array : $h(c, \theta)$.

Let the c be an integer $c \in [0, D]$ where D is the "diagonal distance of the image.
Let θ be an integer $\theta \in [0, 179]$

Algorithm:
```
    allocate a table  h(c, θ) initially set to 0.
    For each x, y of the image
        for  θ from 0 to 179
            c = –x cos(θ) – ysin(θ)
            h(c, θ) = h(c, θ) + E(x, y)
        End
    End
```

The resulting table accumulates contrast.
Peaks in $h(c, \theta)$ correspond to line segments in the image.



Because we know θ(x, y), we can limit the evaluation to θ(x, y)+/- Δθ

5.1 Generalisation of the Hough Transform

We can represent a circle with the equation:

$$(x - a)^2 + (y - b)^2 = r^2$$

We can use this to create a Hough space h(a, b, r) for limited ranges of r.

The ranges of a and b are the possible positions of circles.

Algorithm

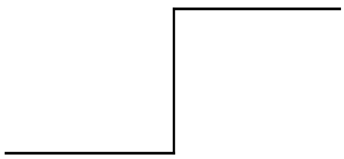
Algorithm:

```
allocate a table  h((a, b, r) initially set to 0.
For each x, y of the image
    for r from r_min to r_max
        for a from 0 to a_max
            b = –y–sqrt( r^2 – (x - a)^2)
            h(a,b,r) = h(a,b,r) + E(x,y).
        End
    End
End
```

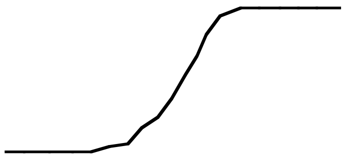
6 Second Derivatives.

An alternative to the gradient is to detect edges as zero crossings in the second derivative.

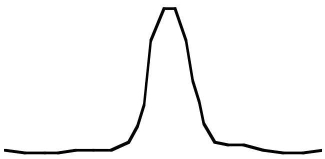
Contrast :



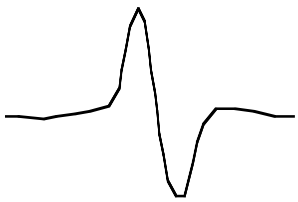
Smoothing :



1st derivative



Second derivative



6.1 Integer Coefficient Second Derivatives

The second derivative is a form of Laplacian operator:

Laplacien: $\nabla^2 p(i,j) = \frac{\Delta^2 P(i,j)}{\Delta i^2} + \frac{\Delta^2 P(i,j)}{\Delta j^2}$

$\Delta_i^2 = [\,1\ \ -1] * [1\ \ -1] = [\,-1\ \ 2\ \ -1]$

$\Delta_i^2 = \boxed{-1\ 2\ -1}$ $\qquad \Delta_j^2 = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$

Laplacian : $\nabla^2 p(i,j) = \Delta_i^2 * p(i,j) + \Delta_j^2 * p(i,j)$

There are several possible discrete forms:

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & -1 \end{bmatrix}$$

$L(u,v) = 4 - 2\cos(u) - 2\cos(v)$

The best is :

$$\begin{bmatrix} -1 & 0 & -1 \\ 0 & 4 & 0 \\ -1 & 0 & -1 \end{bmatrix} + 2 \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -2 & -1 \\ -2 & 12 & -2 \\ -1 & -2 & -1 \end{bmatrix}$$

Gradient : $\| \nabla p(i, j) \| = \sqrt{\left(\frac{\partial P(i,j)}{\partial i}\right)^2 + \left(\frac{\partial P(i,j)}{\partial j}\right)^2}$

Laplacien : $\nabla^2 p(i,j) = \frac{\partial^2 P(i,j)}{\partial i^2} + \frac{\partial^2 P(i,j)}{\partial j^2}$

6.2 Zero Crossings in the second derivative.

In theory

1) Zero crossings give closed contours
2) Zero crossings can be easily interpolated for high precision.

In practice

Zero crossings detect many small unstable contours.

Neighborhood test:

	n_1	
n_2	p	n_0
	n_3	

$$C(i,j) = \begin{cases} 1 & if(sign(n_0) \neq sign(n_2))\ and\ |n_0 n_2| > 0 \\ & or\ \text{if}\ (signe(n_1) \neq signe(n_3))\ and\ |n_1 n_3| > 0 \\ 0 & \text{Otherwise} \end{cases}$$

alternatively :

$$C(i,j) = \begin{cases} 1 & \text{if } (\text{sign}(n_0) \neq \text{sign}(n_2)) \\ & \quad \text{and } |n_0\, n_2| > 0 \\ & \quad \text{and } |n_0 - n_2| > \text{Threshold} \\ & \text{or if } (\text{sign}(n_1) \neq \text{sign}(n_3)) \\ & \quad \text{and } |n_1 n_3| > 0 \\ & \quad \text{and } |n_1 - n_3| > \text{Threshold} \\ 0 & \text{Otherwise} \end{cases}$$

7 Image Description Using Gaussian Derivatives

7.1 Gaussian Derivatives Operators

The Gaussian Function is $G(x,\sigma) = e^{-\frac{x^2}{2\sigma^2}}$

The Gaussian function is invariant to affine transformations.

$T_a\{G(x, \sigma)\} = G(T_a\{x\}, T_a\{\sigma\})$

Recall from lesson 2 we saw that $x_r = x_c \frac{F}{z_c}$

The apparent size of an object is inversely proportional to its distance

A change in size (or scale) is a special case of an affine transform:

$T_s\{G(x, \sigma)\} = G(T_s\{x\}, T_s\{\sigma\}) = G(sx, s\sigma)$

This is just one of the many interesting properties of the Gaussian function.