# Intelligent Systems: Reasoning and Recognition

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# Support Vector Machines with Radial Basis Functions

A radial basis function (RBF) is a real-valued function whose value depends only on the distance from the origin. The Gaussian function

$$f(\|\vec{x}-\vec{c}\|) = e^{-\frac{\|\vec{x}-\vec{c}\|^2}{2}}$$

is a popular Radial Basis Function, and is often used as a kernel for support vector machines. When used in this way, each center point, $\vec{c}$, is one of the support vectors.

We can use a sum of N radial basis functions to define a discriminant function, where the support vectors are drawn from the M training samples. This gives a discriminant function

$$g(\vec{X},\vec{w}) = \sum_{m=1}^{M} a_m y_m f(\| \vec{X}-\vec{X}_m \|) + w_0,$$

The training samples $\vec{X}_m$ for which $a_m \neq 0$ are the support vectors.

Suppose that you have two classes and a training data composed of 10 samples, $\{\vec{X}_m\}\{y_m\}$ and that an SVM learning algorithm has provided the weights $\{a_m\}$ as shown below, with *b=0.*

a) Write out the polynomial for the discriminant function $g(\vec{X},\vec{w})$
b) Is the training data separable with this discriminant function?

| m | y | $x_1$ | $x_2$ | $a_m$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 |
| 2 | 1 | 1 | 3 | 0 |
| 3 | 1 | 2 | 2 | 1 |
| 4 | 1 | 3 | 1 | 0 |
| 5 | 1 | 3 | 3 | 0 |
| 6 | -1 | 1 | 5 | 0 |
| 7 | -1 | 3 | 5 | 1 |
| 8 | -1 | 5 | 1 | 0 |
| 9 | -1 | 5 | 3 | 1 |
| 10 | -1 | 5 | 5 | 0 |