# Intelligent Systems: Reasoning and Recognition

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ENSIMAG 2 Final Exam – 2 July 2021

**Règles et Conditions**

Il est interdit de communiquer avec tout personne autre que le Professeur Crowley entre le moment que vous commencer cette examen et le moment que vous rendez vous solutions par courrier électronique.

Vous avez le droit d'utiliser toutes notes, documents écrits ou documents trouvés en ligne, mais il faut citer toutes vos sources. Vous êtes encouragé à utiliser les documents disponibles sur le site Web du cours. Vous pouvez répondre aux questions en anglais ou en français, mais vous devez illustrer vos réponses avec des mathématiques et des dessins, le cas échéant.

Votre examen terminé doit être soumis en un seul fichier .pdf nommé <VotreNomDeFamille>-ENSI2-SIRR-EXAM.pdf et envoyé par courrier électronique à James.Crowley@grenoble-inp.fr.

Vous pouvez utiliser un logiciel d'édition tel que LaTex ou MS Word, mais sachez que la plusieurs des questions nécessitent l'écriture de mathématiques. Vous pouvez également écrire vos réponses sur papier et envoyer une copie numérisée ou photographiée au format .pdf. Les copies numériques, ainsi que vos réponses écrites, doivent être claires et lisibles.

Rédigez et signez l'attestation suivante à la fin de votre examen:

Je, <votre nom complet>, certifie que je n'ai pas communiqué avec une autre personne ni été aidé par une autre personne pour compléter cet examen. Je reconnais que toutes infraction à cette condition constituerait une violation des règles d'intégrité académique de Grenoble INP et pourrait être passible de sanctions, y compris d'éventuels échec de l'examen ou expulsion.

**Rules and Instructions**

You may not communicate with any person by any means while completing this exam.

You have the right to use any notes, written material or on-line material. You are encouraged to use the documents available on the course web site. You may answer questions in English or in French, but you must illustrate your answers with mathematics and drawings when appropriate.

Your completed exam should be submitted as a single .pdf file named <YourFamilyName>-ENSI2-SIRR-EXAM.pdf and sent by email to James.Crowley@grenoble-inp.fr.

You may use a document typesetting program such as LaTex or MS Word, but beware that some questions require writing mathematics. Alternatively you may write out your answers on paper and send a scanned or photographed copy as a .pdf. Your written answers must be clear and legible.

Write out and sign the following attestation at the end of your exam: "I, <your full name>, certify that I have not communicated with, or been assisted by, any other person in completing this exam. I acknowledge that violation of this condition would constitute a violation of the academic integrity rules of the Grenoble INP and could be subject to penalties, including possible failure or expulsion."

EXAM Questions

1) (4 points) What is an ROC curve? How is it computed? What does it tell you about a classifier? How can you use it to provide an objective comparison of two different classifiers?

2) (4 points) Provide an intuitive explanation for Baye's rule. Under what circumstances can Baye's rule be used? Under what circumstances will Baye's rule give incorrect results?

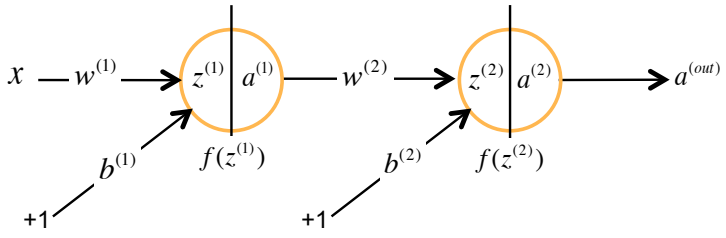
3) (6 points) You are responsible for an International Masters program. You wish to use the academic results of admitted students from previous years to predict the grade point average (GPA) for applicants to your program. As training data, you have the name of the home university, the GPA at the home university and the GPA in your program for a population of students in past years of your program. GPAs are rounded to integer scores in the range 0 to 20.

a) Explain how to use histograms to predict the most likely GPA in your program from the identity and GPA at the home university for an applicant. Give the formulas for estimating the most likely GPA. What is the probability of error for this prediction? How is the accuracy of the prediction affected by the number of prior students?

b) Explain how to use a Normal probability density function to predict the most likely GPA in your program from the identity and GPA at the home university for an applicant. Give the formulas for estimating the most likely GPA as well as for estimating the mean and covariance using the data from past students.

4) (2 points) You have been hired to write a program to provide the fastest route through a subway (metro) network. You decide to use Nilsson's GRAPHSEARCH algorithm. What cost function would you propose? Can you propose a heuristic for which the search is optimal? If yes, provide the conditions for optimality. If no, explain why not.

4) (4 points) Back propagation is a distributed form of Gradient Descent. Consider the following two-layer network with one neural unit per layer. The network has one input variable, $X$, and one output activation, $a^{(out)}=a^{(2)}$.




$$\vec{w} = \begin{pmatrix} w^{(1)} \\ b^{(1)} \\ w^{(2)} \\ b^{(2)} \end{pmatrix} \qquad \nabla C = \frac{\partial C}{\partial \vec{w}} = \begin{pmatrix} \Delta w^{(1)} \\ \Delta b^{(1)} \\ \Delta w^{(2)} \\ \Delta b^{(2)} \end{pmatrix}$$

The error for using these parameters to classify an input sample $X$ with indicator variable $y$ is

$$\delta^{(out)} = \left(a^{(out)} - y\right). \qquad \text{The cost (or Loss) for this error is } C = \frac{1}{2}\left(a^{(out)} - y\right)^2.$$

a) Write the equations for $z^{(1)}$, $a^{(1)}$, $z^{(2)}$ , $a^{(2)}$.

b) Show that $\delta^{(2)} = \delta^{(out)} \dfrac{\partial f(z^{(2)})}{\partial z^{(2)}}$

c) Show that $\Delta w^{(2)} = \delta^{(2)} \cdot a^{(1)}$

d) Show that $\Delta b^{(2)} = \delta^{(2)}$