Dynamic Modeling of Free-Space for a Mobile Robot

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1. Introduction

Free-space is the collection of postures which a vehicle can safely assume. In order to maneuver, a mobile robot requires some description of the limits to free-space. In the paper [Crowley 89] we describe a system which dynamically models free-space of a static environment. This paper describes the extension of that system to model a dynamically changing environment.

This introductory chapter describes the purpose and nature of the process of modeling the geometry of the local environment. It then describes the contents of the paper.

1.1 The Role of Perception in Navigation

Perception serves two fundamentally important roles for navigation:

1) Detection of the limits to free space, and
2) Position estimation.

Free space determines the set of positions and orientations that the robot may assume without "colliding" with other objects. Position estimation permits the robot to locate itself with respect to goals and knowledge about the environment.

Unfortunately, the sensing devices which are available for mobile robots often fail in a variety of circumstances. This is especially true of the less expensive devices such as ultrasonic and infrared range sensors. Combining data from several sensors and from a pre-stored model of the domain provides a way to enhance the reliability of a perception system. Such combination may be accomplished by integrating range measurements into a geometric model of the local environment.

In this paper we describe a technique for dynamically modeling the environment of the robot using ultrasonic sensors and a pre-stored world model. We call such a model a "composite local model. Such a model is "composite" because it is composed of several points of view and (potentially) several sources. The model is local, because it contains only information from the immediate environment. The major topic of this paper is the maintenance of such a model.

An earlier version of this system was described in a recent conference paper [Crowley 89]. This paper describes extensions which are currently under development in that system.

The first improvement is the addition of a "type" field to segments in the composite model. The type of a segment, determined from the pre-stored global model, allows the system to discriminate and react differently to different kinds of surfaces. The type field allows the system to restrict position correction to segments which are known to be stationary. It also permits the system to detect and following segments which are moving.

A second improvement is the inclusion of a new class of geometric primitives in the composite model: points. A point is derived from a sonar reading which does not contribute to a model segment. Points mark limits to free-space which must be respected in free path and find path operations.

The third improvement is the addition of a temporal derivative estimate to the parameter vectors to the composite model primitives. Temporal derivatives allow the tracking of moving objects in the environment of the robot.

The first derivative estimate makes it possible to track moving objects within the model.

1.2 The Perception Process

Modeling the environment is a cyclic process composed of three phases, as illustrated in figure 1.1. These three phases are: Match, Update, and Predict.

In the "match" phase, the new information from perception or from a pre-stored global model is brought into correspondence with the composite model. This correspondence is used in the "update" phase to refine the contents of the model. This refinement includes increasing or decreasing the contents of elements of the model, extending the model to include new information, and removing elements from the model because of a low confidence or because the element is no longer relevant to the goals of the system.

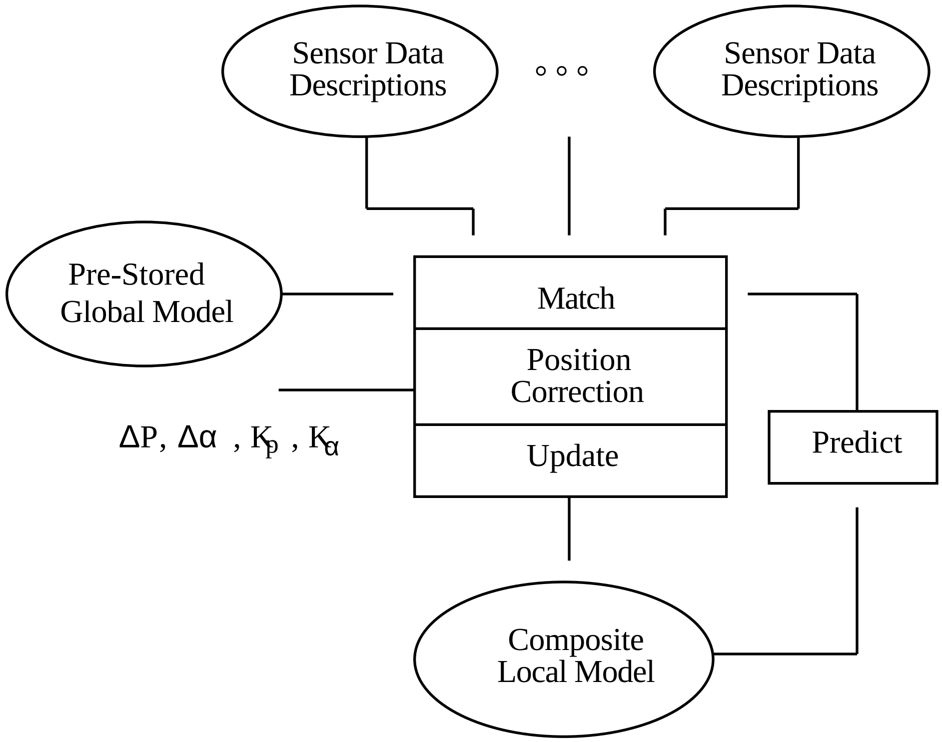

Figure 1.1 Maintaining a world model is a cyclic process

This cyclic process applies to both symbolic and geometric descriptions of the environment. We have implemented and experimented with such a cycle in our early work in world modeling of 2-D free-space with ultrasound [Crowley 85], in a system for modeling a 3-D environment in terms of surfaces and contours [Crowley 86], an in a system for measuring image motion by tracking edge lines [Crowley et. al. 88].

In our earlier work on dynamic modeling, the environment was assumed to be static, and the prediction phase was ignored. However, our recent work in image motion measurement has shown the importance of the prediction phase of the cycle. This paper describes how that lesson is incorporated into the world modeling system for our mobile robot, to provide the ability to discriminate, track and react to moving objects in the environment.

One of the aspects of this paper which is new with regard to previous papers is the manner in which the pre-stored global model is integrated into the system. This global model is created by hand using a interactive graphics tool. The segments in this model carry a label which influences the way in which elements in the model are used.

1.3 Contents of this Paper

The following section reviews our model for the ultrasonic sensor data which is produced by our robot vehicle. Section three presents a representation for line segments in terms of a set of parameters. Each parameter is composed of an estimate and its uncertainty. Such a representation simplifies line segment matching and model updating, particularly in the presence of noise.

Section four reviews our technique for extracting parametric line segments from adjacent, co-linear range measurements. Section five presents the techniques for predicting the position of elements of the model and matching observations to the model. Section six shows the use of a Kalman filter to update the position and orientation of a vehicle from the match of an observed or a recalled segment to the model. Section seven describes the processes for updating the composite local model with an observed segment.

2. Modeling Ultrasonic Range Data

The techniques described below are being implemented using a rectangular mobile robot equipped with a ring of 24 ultrasonic range sensors. Each of the four sides of the vehicle carries a set of three parallel sensors. Each corner also carries a set three sensors mounted with orientations of 30°, 45° and 60°. Range data from all 24 sensors are acquired continuously acquired and projected to "points" in world coordinates, accompanied by an uncertainty. This raw range data is used as the basis for reflex level obstacle detection and avoidance.

The position and orientation of the sensors with respect to the origin of the robot are defined in a sensor configuration table. For each sensor, the sensor configuration table gives:

- r — The distance from the robot's origin to the sensor.
- γ — The angle from the robot's axis to the sensor
- β — The orientation of the sensor with respect to the robot's axis.

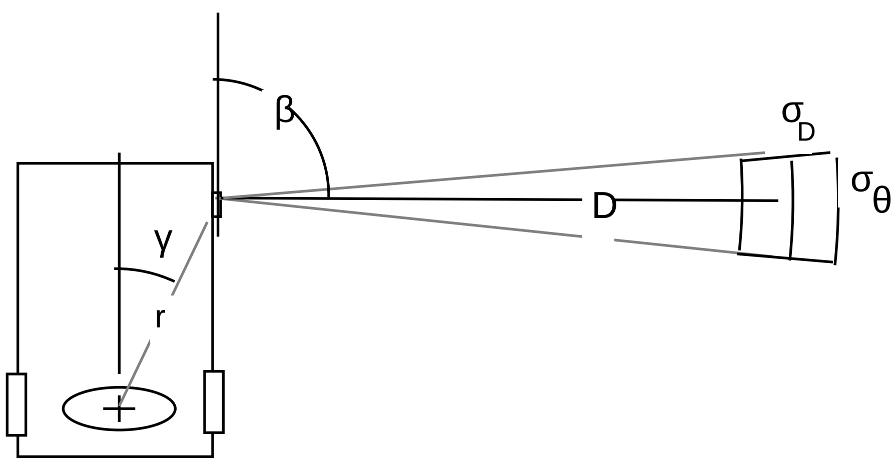

Figure 2.1 Projection of a Range Reading to External Coordinates

A sensor data description process reads range measurements from the sonar table, as well as the estimated position of the robot from the vehicle controller. With this information, the depth measure, d, for each sensor, s, is projected to external coordinates, (x_s, y_s), using the estimated position of the robot, (x, y, α), as shown in figure 2.1.

$$x_S = x + r\ \mathrm{Cos}(\gamma + \alpha) + d\ \mathrm{Cos}(\beta + \alpha)$$
$$y_S = y + r\ \mathrm{Sin}(\gamma + \alpha) + d\ \mathrm{Sin}(\beta + \alpha)$$

In order to combine data from different viewpoints and sensors, we must estimate the inherent precision of the data. We have developed a model of an ultrasonic range sensor which predicts that an echo comes from an arc shaped region defined by an uncertainty in orientation, σ_W, and an uncertainty in depth σ_D.

However, the essential criteria for modeling the sensor data uncertainty is that the estimate of the uncertainty be larger than any true errors. To simplify the data interpretation process we approximate the uncertainty of the measure by a circular variance given by the parameter $\sigma_W{}^2$. In our system, this parameter has been determined, by calibration, to be given by the formula:

$$\sigma_W = 0.10 + d\ \tan(5°)\ \text{(in meters)}$$

Thus each reading in the sonar horizon is represented by the triple (x_S, y_S, σ_{WS}) expressed in external coordinates.

3. Representing Geometric Information

The modeling process begins by constructing a description of the raw sensor data. This description serves to filter sensor noise by detecting range measurements which are mutually consistent. It also provides a representation with which the estimated position and orientation of the robot may be constrained. Finally, it provides a form of "object constancy" at the level of the geometric description of the environment. Such object constancy makes it possible for the mobile vehicle to react to a number of situations without requiring a symbolic interpretation of the model.

Two principles which are basic to our system for world modeling are:

1) The use of a parametric representation for geometric information. In such a representation, each element of the model is represented by a list of parameters, each composed of an estimate and a precision.

2) The use of a common vocabulary for expressing geometric information.

This vocabulary makes it possible to match and integrate geometric information from different sources.

The first part of this section describes a common representation for the limits to free space: a parametric line segment. This is followed by a description of the additional fields used in the composite model.

3.1 A Parametric Representation for Line Segments

Raw ultrasonic range data, the composite local model and the pre-stored global model are each described as parametric line segments [Crowley-Ramparany 87], represented with a data structure, illustrated in figure 3.1. This parametric representation contains a number of redundant parameters which are useful during matching and updating.

A parametric line segment is a structure composed of a minimal set of parameters and a set of redundant parameters. The minimal set of parameters are:

c:	The perpendicular distance from the segment to the origin.
d:	The distance from the perpendicular projection of the origin to the mid-point of the segment.
θ:	The orientation of the line segment.
h:	The half-length of the line segment.
σ_θ:	The uncertainty (standard deviation) in the orientation.
σ_c:	Uncertainty in position perpendicular to line segment.

The redundant parameters for a line segment are:

P:	Mid-point of the line segment in external coordinate (x, y).
P_r:	The end-point to the right of the segment.
P_l:	The end-point to the left of the segment.

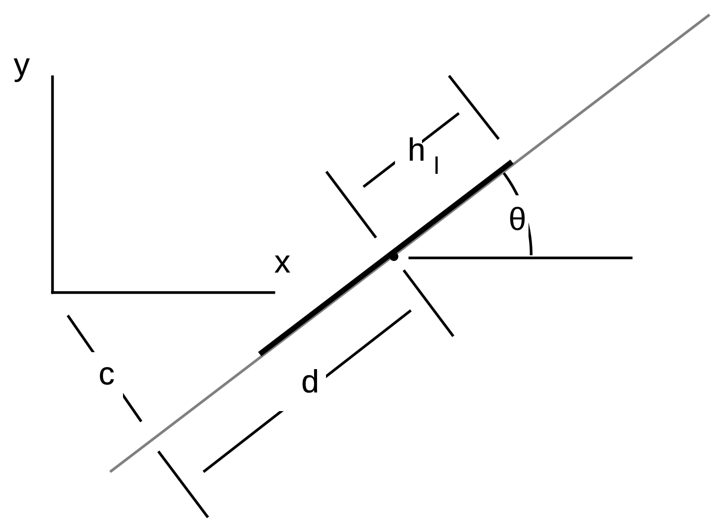

Figure 3.1 The Parametric Representation for a Line Segment.

The parameters (c, d) are equivalent to rotating the segment by an angle of $-\theta$ about the origin so that the midpoint lies on the x axis. That is:

$$d = x \, \mathrm{Cos}(\theta) - y \, \mathrm{Sin}(\theta)$$
$$c = x \, \mathrm{Sin}(\theta) + y \, \mathrm{Cos}(\theta)$$

The advantage of this representation is that it allows us to represent each parameter and its time derivative as a scalar estimate and a scalar variance in the composite model.

In addition to the above parameters, line segments contain a "type" field. Type fields may contain one of the following values:

Observed:	The segment was generated uniquely by observation and does not correspond to a segment in the global model.
Fixed:	A segment from the global model which is known to be unmovable. The label "fixed" is used to represent walls and heavy furniture.
Movable:	A segment from the global model which is known to be movable. Typically used for light furniture which might be displaced in the environment.

The type field is included on both recalled and observed line segments so that the update process can propagate this label to the composite model segments. It is expected that the vocabulary of types will evolve with the robot's functionalities.

3.2 Representation for Line Segments in the Composite Model

Segments which are recalled or observed have no temporal component. Thus, by default, the temporal derivatives are zero and are not explicitly represented. In order to track segments in the composite model, it is necessary to be able to predict the location of the segments at a specific time interval. Such tracking requires an estimate of the temporal derivative of the position and orientation of segments.

Let us express a minimum set of parameters for a segment as the vector

$$\mathbf{S} = [c, d, \theta, h].$$

The parameters of **S** are each represented by a vector **A**, and its covariance, $\mathbf{C_a}$. The vector **A** is composed of an estimate of the parameter, a, plus the estimate of the first temporal derivative, a'. The covariance $\mathbf{C_a}$ composed of the variance of the estimate, σ_a^2, the covariance between the estimate and its temporal derivative, $\sigma_{aa'}$, and the variance of the temporal derivative, $\sigma_{a'}^2$.

$$A = \begin{bmatrix} a \\ a' \end{bmatrix} \qquad C_A = \begin{bmatrix} \sigma_a^2 & \sigma_{aa'} \\ \sigma_{aa'} & \sigma_{a'}^2 \end{bmatrix}$$

where

$$a' = \frac{\partial a}{\partial t}$$

The confidence of existence of segments is represented by a set of confidence states, noted CF, and labeled by the integers 1 through 5. Currently a very simple rule is used for estimating the confidence of segments. When a segment is first added to the composite model, it is entered with CF = 1. When a new segment is found to match a model segment, the CF of the model segment is increased by 1 to a maximum of 5. At the end of each sonar scan, the confidence of all segments is reduced by 1. A segment for which the confidence is smaller than 0 is removed from the model.

Segments are also labeled with a unique "ID". The ID of a segment provides a label by which segments may be "referenced" by processes outside of the modeling cycle.

In summary, the non-redundant parameters of a segment in the composite model are:

Perpendicular Position:	$[\ c,\ c',\ \sigma_c^2,\ \sigma_{cc'},\ \sigma_{c'}^2\ ]$
Tangential Position:	$[\ d,\ d',\ \sigma_d^2,\ \sigma_{dd'},\ \sigma_{d'}^2\ ]$
Orientation:	$[\ \theta,\ \theta',\ \sigma_\theta^2,\ \sigma_{\theta\theta'},\ \sigma_{\theta'}^2\ ]$
Half Length:	$[h,\ h',\ \sigma_h^2,\ \sigma_{hh'},\ \sigma_{h'}^2]$
Type:	One of {Observed, Fixed, Movable}
Confidence Factor:	CF
Identity:	ID

4. Finding Line Segments in Noisy Range Data

Ultrasonic range data are seriously corrupted by reflections and specularities. To overcome these effects we use redundancy. The first source of redundancy is alignment. Reflected readings rarely align. Thus points that are aligned are likely to correspond to actual surfaces. A second source of redundancy is mobility. As the observation position changes, depth measurements based on reflections project to widely varying surfaces, while measurements corresponding to a surface project to that surface. Updating the model from different viewpoints provides a technique by which correct range measures reinforce each other, while measures based on reflections do not.

Line segments are formed in the external coordinate system of the robot using the model of the uncertainty of the ultrasonic sensors described above. Forming line segments in external coordinates permits the integration of range measurements while the robot is moving. The uncertainty of the robot's position is not used in forming line segments because this error is common to the depth measurements. After a segment has been detected and formed, the uncertainty of the robot's position is added to the segment.

This section reviews the process by which line segments are extracted from ultra-sonic range measurements. The presentation is limited to a brief summary based on the more detailed presentation given in [Crowley 89].

4.1 Detecting Segments in Noisy Range Data

To avoid defining line segments with erroneous data, a line segment is not formed unless three consecutive depth readings align within a tolerance. The tolerance is provided by the uncertainty parameter σ_W of the range measurement. The parameters c and θ of the segment computed using weighted averaging, computed using a very simple form which is equivalent to a Kalman filter [Durrant-Whyte 87].

The line segment creation process is applied successively to the sensors in counter-clockwise order. If it succeeds in detecting a segment, then successive points in counter clockwise order are tested to see if they belong to the segment.

4.2 Extension of Segments

For point (x_j, y_j) to be included in a line segment, it must be within a threshold distance of the end point if the segment, and its perpendicular distance from the segment must be less than or equal to half of the uncertainty, σ_{Wj}. If this test is passed, the segment parameters c and θ are updated using the same simple form of Kalman filter, and the segment half length and mid-point are extended to include the point.

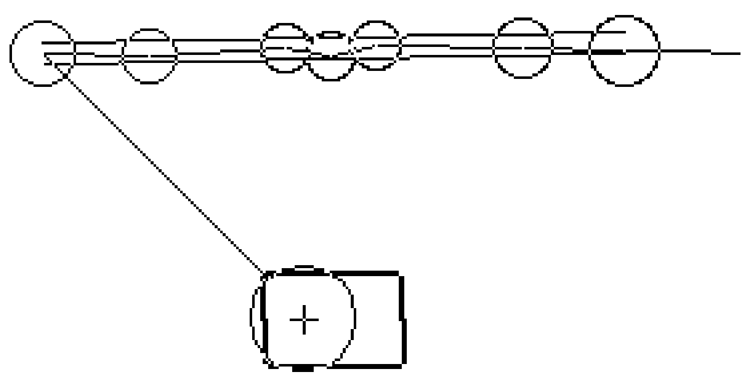

Figure 4.2 A vehicle (rectangle with cross) with an circular uncertainty in position of 40 cm (circle surrounding cross) is shown detecting a line segment. The top image shows a set of range data in which a line segment has been detected. The projections of ultrasound readings are illustrated as circles of radius σ_W. The projections provide the vertices of the sonar horizon. The detected segment is illustrated by a pair of parallel line at $\pm\, \sigma_C$.

The process halts when a point fails the test for inclusion in a segment. When the process halts, it returns the parameters of the segment. Figure 4.2 illustrates this process with crop from a screen dump from an execution of the program.

4.2 Adding the Uncertainty of the Robot to a Segment

The projection of a segment into the external coordinate system is based on the estimate of the position of the vehicle. Any uncertainty in the vehicle's estimated position must be included in the uncertainty of the segment before matching can proceed. This uncertainty affects both the position and orientation of the line segment. The position uncertainty of the segment is increased by computing the component of the robot's uncertainty in the direction of the uncertainty σ_c, that is perpendicular to the segment. For a line segment of orientation θ, the normal vector **M** is

$$\mathbf{M} = [\mathrm{Sin}(\theta), \ -\mathrm{Cos}(\theta)]^T$$

This uncertainty σ_{cr} is determined from the product of the vector and the covariance of the robot's estimated position.

$$\begin{aligned}\sigma_r^2 &= \mathbf{M}^T \mathbf{C}_{xy} \ \mathbf{M} \\ &= \mathrm{Sin}(\theta)^2 \sigma_x^2 - 2 \ \mathrm{Sin}(\theta) \ \mathrm{Cos}(\theta) \ \sigma_{xy} + \mathrm{Cos}(\theta)^2 \sigma_y^2\end{aligned}$$

This is added to the perpendicular uncertainty of the segment by

$$\sigma_c^2 = \sigma_c^2 + \sigma_r^2$$

The uncertainty in orientation of the robot, σ_α^2, is added to the segment by

$$\sigma_\theta^2 = \sigma_\theta^2 + \sigma_\alpha^2$$

This is illustrated in figure 4.3.

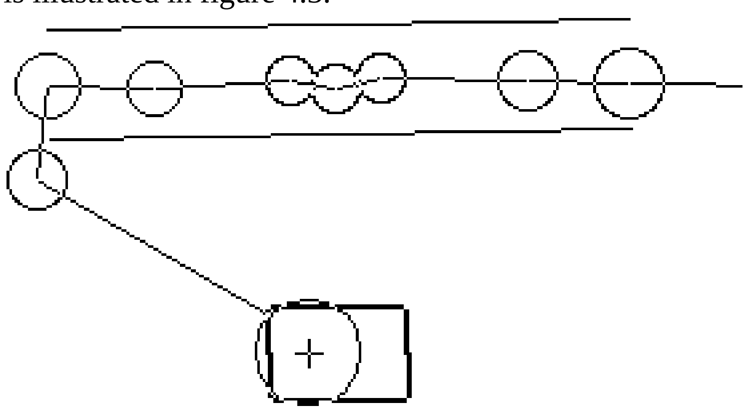

Figure 4.3 This image shows the segment after the uncertainty in the robot's position has been added to the segment uncertainties, σ_c^2 and σ_θ^2.

As each segment is obtained from the ultrasound data it is immediately matched to the composite model, and used to update the composite model and the estimated position of the robot.

However, before matching can occur the state of the model must be predicted for the time interval during which the ultrasound data was obtained.

5. Predicting the State of Model Line Segments

This prediction phase is performed asynchronously at a regular time intervals. The period of between updates is a parameter, Δt. Prediction updates the parameters for segments of the types "observed" and and "movable" based on a simplified form of the Kalman filter prediction equation.

For each attribute A of $S = \{c, d, \theta, h\}$, the estimate of the attribute and of its covariance at a time t is calculated from the estimate at time $t-\Delta t$ using the matrix ϕ.

$$\phi = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}$$

The estimate of each attribute is calculated by:

$$A_t = \phi \; A_{t-\Delta t}$$

The covariance for the attribute at time t, $\mathbf{C}_t$, is calculated by:

$$\mathbf{C}_t = \phi \; \mathbf{C}_{t-\Delta t} \; \phi^T$$

By carrying out the multiplication by hand, one can see that the new estimate is given by:

$$a_t = a_{t-\Delta t} + \Delta t \frac{\partial a_{t-\Delta t}}{\partial t}$$

and its variance is:

$$\sigma^2_{a,t} = \sigma^2_{a,\, t-\Delta t} + 2\,\Delta t\, \sigma_{aa',\, t-\Delta t} + \Delta t^2\, \sigma^2_{a',\, t-\Delta t}$$

This prediction assumes that there is no acceleration between update cycles. Indeed, accelerations exists and are a real source of uncertainty. To account for the possibility of accelerations, the uncertainty of each attribute is increased by an acceleration constant, σ_{acc}^2, multiplied by the time interval to the fourth power. Thus the uncertainty of each attribute for segments of type observed or movable is computed using the formula:

$$\sigma^2_{a,t} = \sigma^2_{a,\, t-\Delta t} + 2\,\Delta t\, \sigma_{aa',\, t-\Delta t} + \Delta t^2\, \sigma^2_{a',\, t-\Delta t} + \Delta t^4\, \sigma^2_{acc}$$

where σ_{acc}^2 is a constant which permits matching despite accelerations in the parameter.

6. Matching Segments to the Composite Model

Matching is a process of comparing a line segment to each of the segments in the composite local model to detect similarity in orientation, co-linearity, and overlap. Each of these tests is made by comparing one of the parameters in the segment representation.

Given an observed or recalled segment, S_o, with parameters $\{x_o, y_o, C_o, h_o, \theta_o\}$ with uncertainties $\{\sigma_{co}, \sigma_{\theta o}\}$ and given a composite model segment, S_m, with parameters $\{x_m, y_m, C_m, h_m, \theta_m\}$ with uncertainties $\{\sigma_{cm}, \sigma_{\theta m}\}$, the comparison begins with a test for similar .

$$(\theta_m - \theta_o)^2 \leq \sigma_{\theta o}^2 + \sigma_{\theta m}^2.$$

If the result of this test is false, the process advances to the next segment in the composite local model. Otherwise, the segments are compared for alignment by testing

$$(c_m - c_o)^2 \leq \sigma_{co}^2 + \sigma_{cm}^2.$$

If this test is true then the two segment are tested for overlap by comparing the distance between center-points to the sum of the half lengths.

$$(x_o - x_m)^2 + (y_o - y_m)^2 \leq h_o + h_m.$$

When a segment in the composite model passes all three tests, it is placed in a list of possible matches. In most cases, the match is unique. If, however, several possible matches are detected, then the potential matching segment are sorted based on similarity measure. This similarity measure is a form of Mahalanobis distance: the sum of the squared differences in parameters normalized by their variances. The normalized distance between two segments is calculated as $Sim_{o,m}$:

$$Sim_{o,\,m} = \frac{(\theta_m - \theta_o)^2}{\sigma_{\theta m}^2} + \frac{(c_m - c_o)^2}{\sigma_{cm}^2}$$

The model segment for which the distance is smallest is selected as a match.

7 Correcting Estimated Position and Orientation

The the cyclic perception process (predict, match and update) permits both observed segments and segments which have been recalled from the global model to be integrated into the composite model in the same manner. The relative uncertainty of segments automatically determines the influence which they have on the estimated position.

When a newly observed segment matches a model segment of type "fixed", the observation provides a provides a one dimensional constraint to the position of the robot, **P**, and its uncertainty $\mathbf{C}_{xy}$, as well as a constraint on the orientation of the robot. In this case, the composite model represents the best estimate of the external world. The position and uncertainty of the observed segment is based on the estimated position and variance of the robot. Thus the robot's position and estimation are corrected by a value which is the observed parameter minus the model parameter. An essential condition for the matching and updating process is that the error in the robot's position and orientation be less than the tolerance region defined by 1 standard deviation.

When a recalled segment of type "fixed" matches a model segment it also provides a constraint on the position and orientation of the robot. In this case, composite model segment is assumed to be corrupted with the errors in the robot's estimated position. In this case, the robot's position and orientation is corrected by the difference of the model parameter minus the recalled parameter.

As the uncertainties in the recalled segment tend to be much smaller or equal to those of the composite model. The importance of the constraint is based on the relative size of the covariances. In both cases, these constraints are applied using a form of Kalman filter update formula [Melsa-Sage 73].

7.1 The Position Constraint Provided by a Segment Correspondence

As before, let us define observed or recalled segment as S_o, with parameters $\{x_o, y_o, c_o, h_o, \theta_o\}$ with uncertainties $\{\sigma_{co}, \sigma_{\theta o}\}$ and the corresponding composite model segment, S_m, with parameters $\{x_m, y_m, C_m, h_m, \theta_m\}$ with uncertainties $\{\sigma_{cm}, \sigma_{\theta m}\}$.

The difference in perpendicular position of two segments, $\Delta c = c_o - c_m$, provides a correction to the estimated position of the robot, as well as to the corresponding segment. The precision of this correction is provided by the variances in perpendicular position from the model and the observed segment, σ_{co}^2 and σ_{cm}^2. This correction applies only in the direction perpendicular to the model segment, defined by the normal vector $M = [\mathrm{Sin}(\theta_m)\ -\mathrm{Cos}(\theta_m)]^T$. For an observed segment, the component of the robot's position uncertainty in this direction is already included in the perpendicular uncertainty of the segment, σ_{co}^2, as described above. When matching a recalled segment, a residual component of the robot's error is assumed to be modeled by σ_{cm}^2.

Thus in either case, a Kalman gain vector for the estimated position of the robot is given by

$$K_p = \mathbf{C}_{xy} \;\; M \;\; 1 / (\sigma_{co}^2 + \sigma_{cm}^2)$$

where $\mathbf{C}_{xy}$ is the covariance inthe robot's position.

The position correction may be given by Δc as described by the gain vector:

$$P = P + K_p \Delta c$$

However, our vehicle controller is designed to accept a correction vector $\Delta P = [\Delta x, \Delta y]$, and a gain matrix $\mathbf{K}_{xy}$. The gain matrix is given by expanding K_p with a cross product with M^T.

$$\mathbf{K}_{xy} = K_p M^T$$

The correction vector is given by $\Delta P = M \Delta c$. The position is then corrected by

$$P = P + \mathbf{K}_{xy} \Delta P$$

Which may be seen as equivalent to $P = P - K_p \Delta c$ by noting that $M^T M = 1$. The vehicle's position covariance is then updated by

$$\mathbf{C}_{xy} = \mathbf{C}_{xy} - \mathbf{K}_{xy} \mathbf{C}_{xy}$$

The vehicle's estimated orientation is updated by computing a Kalman gain, k_α:

$$k_\alpha = \sigma_\alpha{}^2 / (\sigma_{\theta m}{}^2 + \sigma_{\theta o}{}^2)$$

The orientation is then updated by the difference in angle between the model and observed segment:

$$\alpha = \alpha + k_\alpha (\theta_o - \theta_m).$$
$$C_\alpha = C_\alpha - k_\alpha C_\alpha$$

8. Updating the Composite Local Model

Updating the composite model involves aligning the composite model with the observed or recalled segment, and determining the segments type and confidence.

In the case of an observed segment, the position and orientation uncertainties tend to be large relative to the segment in the composite model. In this case the composite model segment is only slightly shifted by the observation. Similarly, a recalled segment of type "movable" has relatively large uncertainties in position and orientation and thus has only a minor influence on the composite model.

On the other hand, a recalled segment of type "fixed" has small uncertainties in position and orientation, determined at the time that the model of the environment was constructed. In this case, the recalled segment exerts a strong influence on the position of the corresponding segment in the composite model.

This section describes the process for updating the composite model. It first presents the process for updating the position and orientation of a segment. It then describes the process for determining the type and confidence of a matched segment.

8.1 Updating the Position and Orientation of a Model Segment

The update process is based on a simplified form of the Kalman filter. Let C_a represent the covariance of the attribute A in the composite model segment and C_o represent the variance of the attribute A in the observed or recalled segment. The general form of the Kalman update Gain is:

$$\mathbf{K} = \mathbf{C_a} \mathbf{H} (\mathbf{H}^T \mathbf{C_a} \mathbf{H} + \mathbf{C_o})^{-1}$$

Where **H** is the transformation of the observation to the model. The observation or recall contains only an instantaneous parameter values. It makes no contribution to the estimate of the derivative. Thus the vector **H** is the vector:

$$\mathbf{H} = [\ 1 \quad 0\ ]^T$$

The term $\mathbf{H}^T \mathbf{C_a} \mathbf{H}$ in the gain formula transforms $\mathbf{C_a}$ to a single variance σ_a^2. This variance is combined with the variance of the observation, σ_o^2. The inverse of this sum is then transformed by the vector **H** to have the form of a vector. This vector is then multiplied by the current covariance to give the Kalman gain vector, K. This gain vector can be expressed as two scalar gain terms, K_a and $K_{a'}$, which give the gain for the parameter and its derivative respectively.

$$K = [K_a, \quad K_{a'}\ ]^T$$

where :

$$K_a = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_o^2} \qquad K_{a'} = \frac{\sigma_{aa'}}{\sigma_a^2 + \sigma_o^2}$$

The state vector, A, is multiplied by the vector H to obtain the estimate of the attribute. The estimate is subtracted from the observed value of a_o to give a correction vector. This correction vector is then multiplied by the Kalman gain vector to obtain the updated value for the attribute vector. Thus the update formula for the attribute vector A (attribute and its derivative) is given by

$$A = A + K [a_o - H^T A].$$

For the attribute estimate and its derivative, this formula is equivalent to

$$a = a + K_a (a_o - a)$$

and

$$\frac{\partial a}{\partial t} = \frac{\partial a}{\partial t} + K_{a'} (a_o - a)$$

In the general case, the covariance of an estimate vector is updated by

$$\mathbf{C_a = C_a - K\, H^T\, C_a.}$$

Evaluation of this formula shows that the variances are updated by

$$\sigma_a^2 = \sigma_a^2 - K_a\, \sigma_a^2$$

$$\sigma_{aa'} = \sigma_{aa'} - K_a\, \sigma_{aa'} = \sigma_{aa'} - K_{a'}\, \sigma_a^2$$

$$\sigma_{a'}^2 = \sigma_{a'}^2 - K_{a'}\, \sigma_{a'}^2$$

If no match is found, the token attributes are updated using the predicted value for each of the attributes. That is $a_o = a$.

Segments of type "fixed" are assumed to not move. Thus these segments have an estimate and covariance given by

$$A = \begin{bmatrix} a \\ 0 \end{bmatrix} \qquad C_A = \begin{bmatrix} \sigma_a^2 & 0 \\ 0 & 0 \end{bmatrix}$$

Thus in this case, $K_{a'}$ remains zero, and the update formula provides only a new value for the parameter and its variance.

8.2 Extending Segments

Structures in an environment are often larger than the robot's field of view. In such a case, any one time the ultrasonic ring will see only a small part of a the surface. This problem is compounded by specularities. Thus the endpoints of the description which has been saved in the global model is assumed to be more reliable than those of the composite model.

When the segment S_0 is of type fixed or movable, its end points replace those of the composite model segment. These endpoints are then projected to the line defined by c and θ. The midpoint, and the parameters d_m and h_m are then computed from the endpoints.

In the case of observed data, the parameters d_m and h_m are updated using the kalman filter update equations given above, and the midpoint and endpoints are then calculated from these parameters.

8.3 Updating the Type and Confidence of Segments.

The recall and match of segments from the globals model is performed by a perception function. This function is called by a higher level supervisor as the robot enters a new "place". The strategy used in this recall is beyond the scope of this paper.

Segment type labels obey a simple precedence rule. In a match between a fixed segment and a segment of any other label, the updated model segment is labeled as type "fixed". For a match of segments if type movable and type observed, the result is type movable. Observed segments retain their label only when matched to observed segments.

8.4 Segments for which no Match is Found

When a segment is matched to the composite model and no correspondence is found, the segment is added to the composite model with a confidence state of CF = 1. If the segment is from the global model and of type "fixed", the temporal derivative and covariances are set to zero. Otherwise, the segment is added to the model using zero for the temporal derivatives, and default values for the terms $\sigma_{aa'}$ and $\sigma_{a'}^2$ of the covariances.

9 Discussion

Two conclusions can be drawn from this system:

1) An explicit model of uncertainty using covariances and Kalman filtering provides a tool for integrating noisy and imprecise sensor observations into a model of the geometric limits to free space of a vehicle.

2) Such a model provides a technique for a vehicle to maintain an estimate of its position as it travels, even in the case where the environment is unknown.

This is the third system which we have constructed using these mathematical techniques. Earlier systems include a technique for combining data from 3-D sensors [Crowley 86] and a technique for measuring the movement of edge lines in motion sequences [Crowley 88]. A number of other authors report similar positive results with these tools. Matthies et al have used similar techniques for motion and stereo [Matthies-Shafer 87]. Durrant-Whyte has used similar techniques for combining touch and stereo [Durrant-Whyte 87]. Faugeras and Ayache have used similar techniques for 3-D modeling using sequences of stereo images [Faugeras-Ayache 86].

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